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# NOMOGRAMS OF ADJUSTMENT.*

BY L. I. HEWES.

D'Ocagne has shown† that any equation of the form

$$(1)\qquad \begin{vmatrix} f_1 & g_1 & 1 \\ f_2 & g_2 & 1 \\ f_3 & g_3 & 1 \end{vmatrix} = 0,$$

where $f_i$ and $g_i$ are any functions of one variable, may be represented by a collinear nomogram or diagram of alignment. For this purpose it is only necessary to set up the three pairs of defining equations‡

$$x = f_i, \quad y = g_i \qquad (i = 1, 2, 3),$$

and plot the three corresponding curves which are inscribed with the values of the corresponding parameters $z_i$. A graphical solution for any unknown $z_i$ is then found by applying a straight edge or index through two known values of $z_i$.

This procedure has also been extended to include equations of the type

$$(2)\qquad \begin{vmatrix} f_{12} & g_{12} & 1 \\ f_{34} & g_{34} & 1 \\ f_{56} & g_{56} & 1 \end{vmatrix} = 0,$$

where the equations

$$x = f_{ij}, \quad y = g_{ij} \qquad (i = 1, 3, 5;\ j = 2, 4, 6)$$

define in general three curve nets.

It is observed that in equations (1) and (2) the variables $z$ are so segregated that any one of them appears in but one row of the determinant. The class of equations to which the theory is applicable is consequently restricted. It is proposed to remove this particular restriction from equation (1), and from equation (2) when the number of variables does not exceed five, by introducing the additional principle of *adjustment of the index.*

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* Read by title at the Cambridge Summer Meeting of the American Mathematical Society, Sept. 5, 1916.

† Traité de Nomographie, Paris, 1899, Ch. III, and Ch. V, and Calcul Graphique et Nomographie, Paris, 1908, Ch. IV.

‡ Indicated by d'Ocagne, l. c., and adopted by Soreau, "Contribution à la théorie . . . de la Nomographie," Mém. et comptes rendus de la Soc. des Ing. civils, 1901.

Consider first equations of the form

$$
\begin{vmatrix} f_{ij} & g_{ij} & 1 \\ f_{kl} & g_{kl} & 1 \\ f_{mn} & g_{mn} & 1 \end{vmatrix} = 0, \tag{3}
$$

where the subscripts denoting the variables are allowed to take on in pairs *any* values from the set of numbers 0, 1, 2, 3 with the understanding that 0 shall denote the absence of a second variable in any function. Upon setting up the defining equations

$$
\begin{aligned}
x &= f_{ij}, & y &= g_{ij}, \\
x &= f_{kl}, & y &= g_{kl}, \\
x &= f_{mn}, & y &= g_{mn},
\end{aligned}
$$

it is seen that the corresponding geometric configuration in the plane, in the most general case, consists of three curve nets and that when any two nets are considered there always appears one family of curves in each in which the curves correspond to the *same* parameter $z$. In general three collinear points found respectively in the three nets will not yield values of the parameters $z$ attached to the corresponding pairs of curves intersecting at the points, which are solutions of equation (3), but six unrelated values of the parameters $z_1$, $z_2$, $z_3$. When however six curves, which are inscribed with values of $z_1$, $z_2$, $z_3$, which constitute a solution of equation (3) are selected in the three nets, it is at once evident from the defining equations of equation (3) that the six curves intersect in pairs at points which are collinear. It remains therefore to establish a method for the converse process.

In practice one variable only is unknown, therefore there will always be one curve net in the most unfavorable case in which a point is determined by the values of the two knowns. In each of the remaining two nets one curve is known. Therefore if the index is rotated about the given point until its intersections with the two known curves are at points on curves inscribed with the *same* value of the unknown, that value of the unknown $z$ is the value sought.

There are eight distinct cases of equation (3) if all trivial cases are excluded. These eight cases have the following determinants for the left-hand member:

$$\begin{array}{c}\text{I.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_2 & g_2 & 1\\ f_3 & g_3 & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{II.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_2 & g_2 & 1\\ f_{13} & g_{13} & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{III.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_{12} & g_{12} & 1\\ f_{13} & g_{13} & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{IV.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_{12} & g_{12} & 1\\ f_{23} & g_{23} & 1\end{vmatrix}\end{array},$$

$$\begin{array}{c}\text{V.}\\ \begin{vmatrix} f_{12} & g_{12} & 1\\ f_{13} & g_{13} & 1\\ f_{23} & g_{23} & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{VI.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_{23} & g_{23} & 1\\ f_{23} & g_{23} & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{VII.}\\ \begin{vmatrix} f_{12} & g_{12} & 1\\ f_{13} & g_{13} & 1\\ f_{13} & g_{13} & 1\end{vmatrix}\end{array}, \quad \begin{array}{c}\text{VIII.}\\ \begin{vmatrix} f_1 & g_1 & 1\\ f_1 & g_1 & 1\\ f_{23} & g_{23} & 1\end{vmatrix}\end{array}.$$

It is seen that in cases III and VII it is possible that the unknown variable may enter in each row in which case no initial fixed point would be determined and the position of the index which yields a solution would only be found by trial and error. On the other hand in these same two cases if the variable entering each row is known there is no adjustment of the index required.[*] Nor is adjustment required in several other cases, *e. g.*, II when $z_3$ is unknown, etc. Case I is of course d'Ocagne's equation of three variables.

In many examples in practice one (or more) of the functions reduces to a constant in which event d'Ocagne's notion of a binary scale[†] is needed in carrying out the construction unless the other function in the corresponding row contains but one variable; in that case only a straight scale is involved. If the unknown value is involved in the net of the binary scale the management of the index becomes troublesome unless a known value enters in each row of the determinant.

As an application of the above theory consider the equation

$$z_1 z_2 - z_3 + \sqrt{1 + z_2^2}\sqrt{1 + z_1^2} = 0,$$

which d'Ocagne has pointed out[‡] cannot be written in the form (1) above. This equation is however of the type VIII of equation (3). It may be written in the determinant form

$$\begin{vmatrix} 1 & z_1 & 1\\ 0 & \sqrt{1 + z_1^2} & 1\\ \dfrac{z_2}{z_2 + \sqrt{1 + z_2^2}} & \dfrac{z_3}{z_2 + \sqrt{1 + z_2^2}} & 1 \end{vmatrix} = 0. \tag{4}$$

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[*] Unless $z_3$ is unknown in case VII.

[†] L. c., Ch. V and Ch. III respectively.

[‡] L. c., pp. 419–22 and pp. 185 and 229 respectively.

The corresponding defining equations are

$$x = 1, \qquad y = z_1,$$

$$x = 0, \qquad y = \sqrt{1 + z_1^2},$$

$$x = \frac{z_2}{z_2 + \sqrt{1 + z_2^2}}, \quad y = \frac{z_3}{z_2 + \sqrt{1 + z_2^2}}.$$

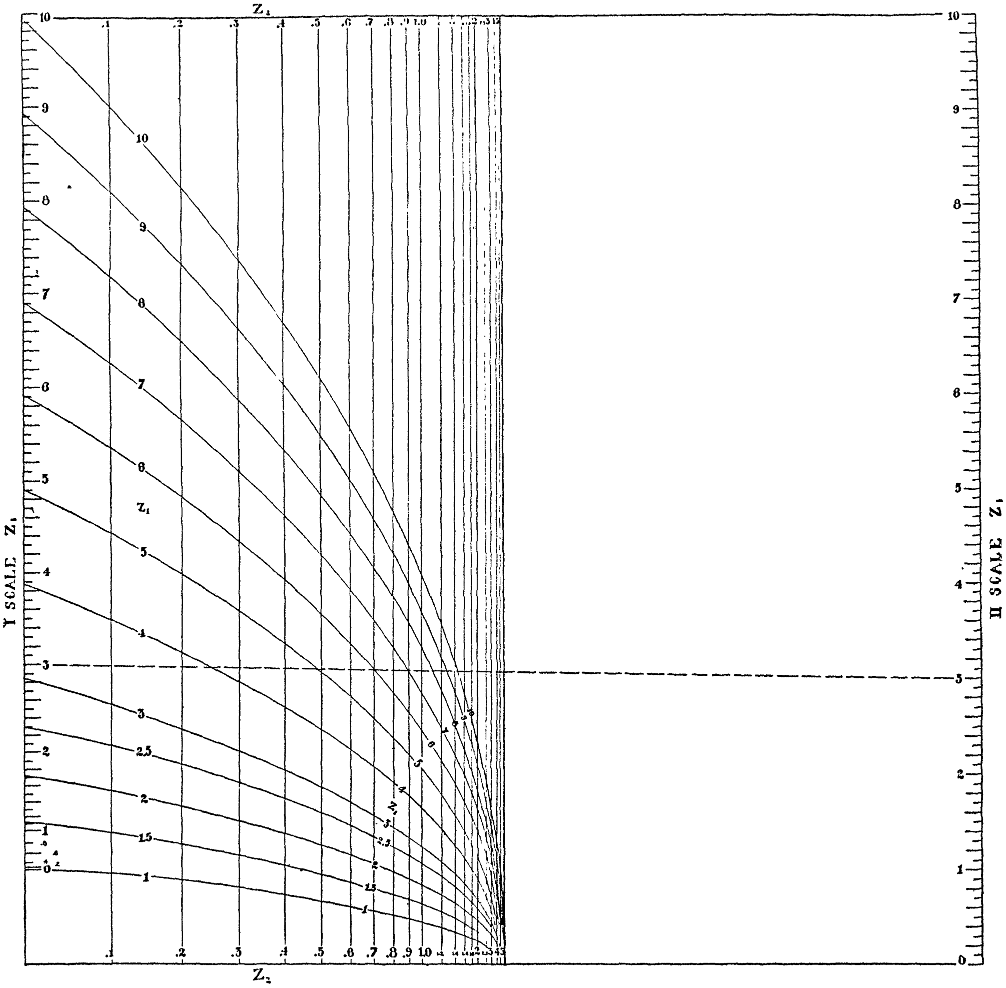

FIG. 1. Nomogram for the Equation $z_2z_3 - z_1 1\sqrt{1+z^2}\sqrt{1+z^2} = 0$.

Since $z_1$ appears only in $g_1$ the first two scales for $z_1$ are straight scales on the lines $x = 1$ and $x = 0$. The net of curves for $z_2$ and $z_3$ consists of lines parallel to the $y$- axis inscribed with values of $z_2$ and the family of parabolas

$$y^2 = z_3^2(1 - 2x)$$

and inscribed with corresponding values of $z_3$. The finished diagram is shown in Fig. 1. If $z_1$ is known, no adjustment of the index is required but if it is unknown then the index must be rotated about the point determined by $z_2$ and $z_3$ in the net until the intersections with the two straight scales yield the same value.

By analogy with the class of equations (3) equations of more than three variables may be represented by nomograms in which corresponding geometric elements appear more than once. The equation

$$\begin{vmatrix} f_{ij} & g_{ij} & 1 \\ f_{kl} & g_{kl} & 1 \\ f_{mn} & g_{mn} & 1 \end{vmatrix} = 0, \tag{5}$$

where the subscripts are allowed to take on any two different values in pairs from the numbers 0, 1, 2, 3, 4, and 5, exhausts all possible cases.

An example of equation (5) is afforded by the bi-quadratic equation

$$z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0,$$

which may be written in the determinant form

$$\begin{vmatrix} -z & a_4 + z^4 & 1 \\ \dfrac{a_2}{a_1} & \dfrac{a_2}{a_1} a_3 & 1 \\ 0 & -a_2 z^2 & 1 \end{vmatrix} = 0,$$

and the quotient of the two coefficients $a_2/a_1$ may be regarded as a single variable parameter $k$. The only calculation involved in the solution is then to determine this quotient. The nomogram is constructed from the defining equations

$$x = -z, \quad y = a_4 + z^4,$$

$$x = \frac{a_2}{a_1}, \quad y = \frac{a_2}{a_1} a_3,$$

$$x = 0, \quad y = -a_2 z^2,$$

and it is necessary to have a binary scale on the line $x = 0$ for which the segregation of the variables $a_2$ and $z$ is accomplished by setting $x = -z$ in order to take advantage of the $z$ lines already required in the first net. The nomogram is entered with the value of the parameter $k$ and the coefficient $a_3$ which determine a point in the second curve net. The index must then be rotated until it intersects the proper $a_4$ curve at a point with the same abscissa as the point of intersection of the parabola

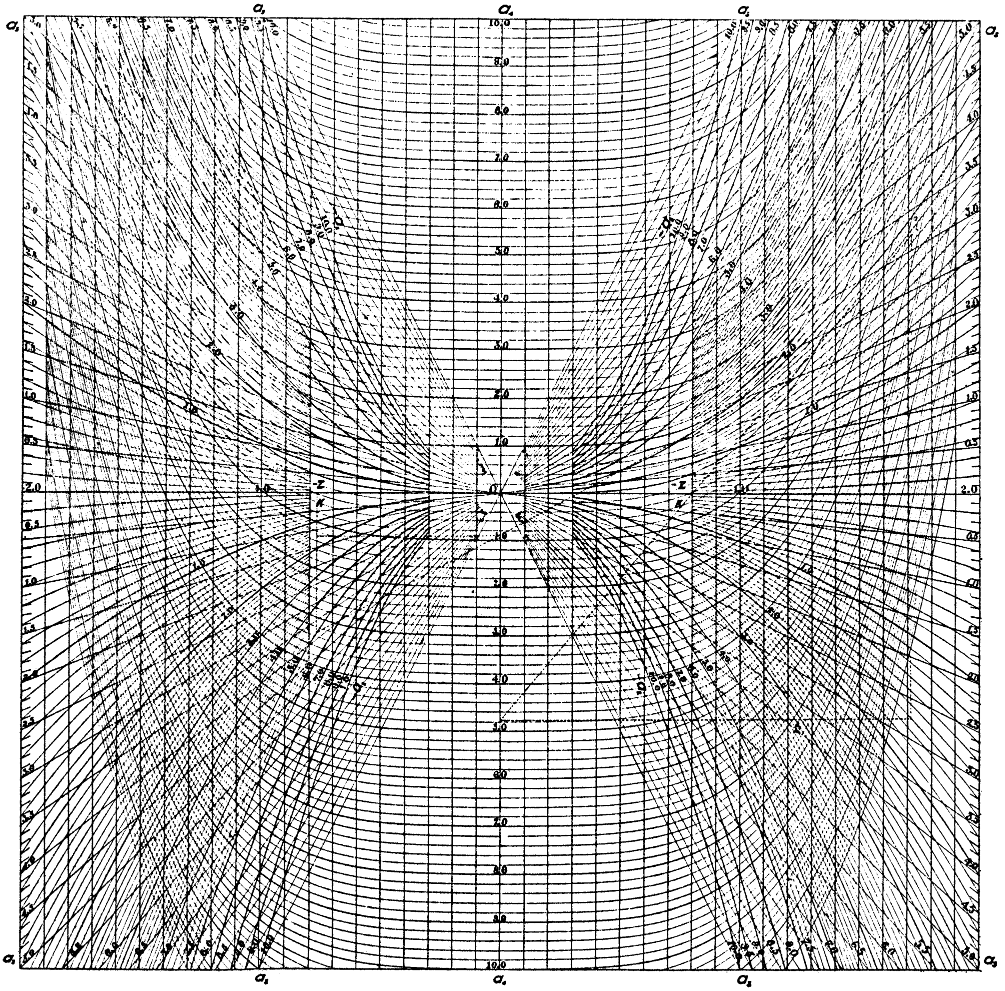

FIG. 2.

$a_2$ and the horizontal projecting line from the point of intersection of the index with the $Y$ axis. In the figure the index is set for the two real roots of the bi-quadratic equation

$$z^4 + 1.33z^3 + 1.6z^2 + 2z - 3.3 = 0.$$

FAIRFAX COUNTY, VA.,
September 9, 1916.